







Euclíd

# MATHEMATICS Winter Number Land

A

Grade 8

## Winter 2011-2012

aiving our students the words

Miami-Dade County Public Schools Curriculum & Instruction

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Mathematics Winter Packet 2011-2012 Grade 8

### WELCOME TO A MATHEMATICS WINTER NUMBER LAND

The realm of mathematics contains some of the greatest ideas of humankind. The *A Mathematics Winter Number Land* activities included in this packet are a mathematical excursion designed to be read, fun to do, and fun to think and talk about. These activities will assist you in applying the concepts you have studied. Additionally, each activity addresses a specific Next Generation Sunshine State Benchmark. Each benchmark is listed at the end of the activity.

The journey to true mathematics understanding can be difficult and challenging but be patient and stay the course. Mathematics involves profound ideas. As we make these ideas our own, they will empower us with strength, techniques, and the confidence to accomplish wonderful things. Enjoy working each activity.

Included as part of this packet, is a link to the Miami-Dade County Public Schools Student Portal *Links to Learning* technology activities. Individualized student learning paths have been designed based on FCAT scores and are aligned to the District's Pacing Guides. These online activities are supplemental and, as such, are not to be assigned or graded. All online activities are provided as a resource to both parents and students to engage learning using technology. Please log on just as you do at your school.

#### Tips for A Mathematics Winter Number Land

Read the activity and attempt to answer the questions that follow. The only rules are:

- 1. Make an earnest attempt to solve the problem. Record your attempts.
- 2. Be creative.
- 3. Don't give up. If you get stuck, look at the story and question a different way.
- 4. Discuss your story with your family.
- 5. HAVE FUN!

If you are in need of additional information about the *A Mathematics Winter Number Land* Winter Break Activity Packet, please contact the Division of Mathematics, Science, and Advanced Academics Programs, at 305 995-1934.

### Who Were They?

**Pythagoras** was a Greek mathematical genius and often described as the first pure mathematician. He invented the Pythagorean Theorem which states that: "In any right triangle, the area of the square whose side is the hypotenuse (the side of a right triangle opposite the right angle) is equal to the sum of areas of the squares whose sides are the two legs (i.e. the two sides other than the hypotenuse)."

*Euclid*, the Greek mathematician, was known as the "Father of Geometry". He taught at the university in Alexandria, Egypt. While at the university, he compiled his famous 13 volume treatise called *Elements* that is still the basis of the geometry taught in schools to this day. He used axioms (accepted mathematical truths) to develop a deductive system of proof, which he wrote in his textbook *Elements*. Euclid's first three postulates, with which he begins his *Elements*, are familiar to anyone who has taken geometry: 1) it is possible to draw a straight line between any two points; 2) it is possible to produce a finite straight line continuously in a straight line; and 3) a circle may be described with any center and radius.

*Euclid* also proved that it is impossible to find the "largest prime number," because if you take the largest known prime number, add 1 to the product of all the primes up to and including it; you will get another prime number. Euclid's proof for this theorem is generally accepted as one of the "classic" proofs because of its conciseness and clarity. Millions of prime numbers are known to exist, and more are being added by mathematicians and computer scientists. Mathematicians since Euclid have attempted without success to find a pattern to the sequence of prime numbers.

**Archimedes** is one of the great scientists of antiquity also known for his mathematical work. It is believed he studied under followers of Euclid. He proved that an object plunged into liquid becomes lighter by an amount equal to the weight of liquid it displaces. Popular tradition has it that Archimedes made the discovery when he stepped into the bathtub, then celebrated by running through the streets shouting "Eureka!" ("I have found it!"). He also worked out the principle of levers, developed a method for expressing large numbers, discovered ways to determine the areas and volumes of solids, and calculated an approximation of pi ( $\pi$ ).

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### **MUSICAL SHAPES**

Pronounced see-dee-rom. Short for Compact Disc-Read-Only Memory, a type of optical disk capable of storing large amounts of data -- up to 1GB, although the most common size is 650MB (megabytes). A single CD-ROM has the storage capacity of 700 floppy disks, enough memory to store about 300,000 text pages.

CD-ROMs are stamped by the vendor, and once stamped, they cannot be erased and filled with new data. To read a CD, you need a CD-ROM player. All CD-ROMs conform to a standard size and format, so you can load any type of CD-ROM into any CD-ROM player. In addition, CD-ROM players are capable of playing audio CDs, which share the same technology.



CD-ROMs are particularly well-suited to information that requires large storage capacity. This includes large software applications that support color, graphics, sound, and especially video.

### **Applying Volume:**

At winter time, music CDs are popular gifts during the holiday season because they are inexpensive. Today's music comes in many forms. You're probably most familiar with compact discs than cassette tapes. Maybe you've seen an old vinyl record or the mini-discs. These forms of music look and play differently, but you have noticed that all areas have the same geometric shape, a circle.

### **MUSICAL SHAPES**

### ACTIVITY PAGE

### Pump up the Volume

### Steps Directions

Gather the following materials: centimeter ruler and at least ten CDs.

- 1. Measure the radius of a CD
  - a. Find the height of a CD (hint: stack several CDs, measure their combined height, and divide by the number of discs)
  - b. Find the volume of the CD
  - Measure the radius from the center of the CD to the beginning of the music area. (see the photo below) This section contains no music.
    Find its volume.
  - d. Subtract your answer to part (c) from your answer to part (b) to find the volume used for music.
- 2. How many minutes of music are stored on a CD?
- 3. How many minutes are stored in each cubic centimeter of volume?



Radius of CD

Which format stores music more efficiently a CD or a DVD? Explain.

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### **MUSICAL SHAPES**

### **BENCHMARK:**

#### MA.8.G.5.1

Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)) and dimensions including temperature, area, volume, and derived units to solve problems.

When people go on winter vacation, they usually stay in a hotel and rent a car. They are charged at a daily rate for both the hotel and car. Suppose the rental charge for the car is \$15.00 per day. The amount a person would have to pay for the car depends on the number of days that they had the car.

Number of days	Rental Charge
1	\$15.00 x 1 = \$15.00
2	\$15.00 x 2 = \$30.00
3	\$15.00 x 3 = \$45.00
4	\$15.00 x 4 = \$60.00

The rental charge follows this pattern:

Rental charge =  $15.00 \times 10^{-1}$  Rental charge =  $15.00 \times d$ 

The letter *d* stands for the number of days shown in the table: 1, 2, 3, or 4. Also, *d* can stand for other hours not in the table. We call *d* a variable.

A **variable** is a symbol used to represent one or more numbers. The numbers are called **values of the variable**. An expression that contains a variable, such as  $15.00 \times d$ , is called a **variable expression**. An expression, such as  $15.00 \times 4$ , that makes a particular number is called a **numerical expression** or **numeral**.

Another way to indicate multiplication is to use a raised dot, for example,  $15.00^{\circ}$  4. In algebra, products that contain a variable are usually written without the multiplication sign because it looks too much like the letter *x* which is often used as a variable.

15 x *n* can be written as 15*n* a x b can be written as ab  $\frac{1}{2}$  x a can be written as  $\frac{1}{2}$  a

The number named by a numerical expression is called the **value of the expression.** Since the expression 4 + 2 and 6 name the same number, they have the same value. To show that these expressions have the same value, you use the equal sign, =. You write

#### 4 + 2 = 6

and say "four plus two *equals* (or *is equal to* or *is*) six." The *simplest,* or most common, name for the number is 6. The symbol ≠ means *"is not equal to.* You write

4+ 2 ≠ 5

To show that the expression 4 + 2 and 5 do not have the same value.

Replacing a numerical expression by the simplest name for its value is called **simplifying an expression**. When you simplify a numerical expression, you use the substitution principle.

### **EXAMPLE 1**

Simplify each expression. **a.**  $(42 \div 6) + 8$  **b.**  $54 \div (8 - 2)$ 

The parentheses () show how the numerals in the expression are to be grouped. The expression within the parentheses is simplified first.

*a.*  $(42 \div 6) + 8 = 7 + 8 = 15$  *Answer* Note that to read the symbols "42 ÷ 6) + 8," you may say "the *quantity* forty-two divided by six, plus eight"

**b.**  $54 \div (8 - 2) = 54 \div 6 = 9$  **Answer** 

Replacing each variable in a variable expression by a given value and simplifying the result is called **evaluating the expression** or **finding the value of the expression**.

### EXAMPLE 2

Evaluate each expression if a = 5. **a.** 7a **b.** (3a) + 2

a. Substitute 5 for a	<b>b.</b> Substitute 5 for a
7a = 7.5	(3 <i>a</i> ) + 2 = (3 · 5) + 2
= 35 <b>Answer</b>	= 15 + 2 = 17 <b>Answer</b>

In example 1 and 2 the parentheses show how the variables and numbers in the expression are to be grouped. Notice that *expressions within parentheses should be simplified first.* 

### **EXAMPLE 3**

Evaluate (5x) - (3 + y) if x = 12 and y = 9.

First replace x with 12 and y with 9, and insert the necessary multiplication symbol. Then simplify the result. (5x) - (3 + y)

 $(5 \cdot 12) - (3 + 9)$ 

60 – 12 = 48 *Answer* 

### Simplify each expression

1.	29 – (0 · 9)	2.	5 – (16 ÷ 4)	 3.	(8 · 17) + (12 ·17)	
4.	(12 · 11) – (2 · 11) _	5.	(24 + 4) ÷ (30 ÷ 2)	 6.	(16 ÷ 4) ÷ (1 <sup>·</sup> 4)	

### Evaluate each expression if x = 2, y = 3, and z = 4.

1.	5 <i>x</i>	 11. <i>xy</i>	-	
2.	(4 <i>x</i> ) + 7	 <b>12.</b> (52	z) — 7	
3.	(2x) + (2y)	 <b>13.</b> 8()	/ + Z)	
4.	$\frac{1}{2}$ (z - x)	 <b>14.</b> $\frac{(y)}{(y)}$	(x+x) (x-x)	
5.	(4 <i>x</i> ) – 8	 <b>15.</b> (5)	/z) – x	
6.	6 <i>y</i>	 <b>16.</b> <i>yz</i>	-	
7.	(3 <i>y</i> ) – 9	 <b>17.</b> (8)	() + 7	
8.	(3z) - (4x)	 <b>18.</b> 7(y	/ – <i>X</i> )	
9.	$\frac{2}{3}$ . (y + 6)	 <b>19.</b> $\frac{(z)}{(z)}$	(x+y) (x-y)	
10.	4 <i>z</i> – (8 <i>x</i> )	 <b>20.</b> 8 +	- (9 <i>xy</i> )	

**BENCHMARK:** 

MA.8.A.6.3 Simplify real number expressions

.

### Algebra

### Adapted from http://education.ti.com/go/NUMB3RS © 2006 Texas Instruments Incorporated

The word "algebra" is derived from the Arabic word al-jabr. This term is found in Mohammed ibn Musa al-Khwarizmi's book *The Comprehensive Book of Calculation by Balance and Opposition*, written around the year 825. Balance is a translation of the word *al-jabr*, which eventually became *algebra*.

In al-Khwarizmi's book, he did not use the modern algebraic notation, neither did he use equations. Instead, everything was in words. For example, he used the Arabic word shay, or thing, in place of X. The text was a manual for solving equations. He mainly dealt with square (of the unknown), roots of the square, and absolute numbers (constants). He noted six different types of quadratic equations, such as squares equal to roots ( $ax^2 = bx$ ) and squares equal to numbers ( $ax^2 = c$ ).

Today, from scuba diving to crime fighting, Algebraic equations are used to solve problems. Fighting crime may not be the first thing that springs to mind when you think of Algebra. But *CSI Miami, Numb3rs* and *NCIS* on CBS television, are just a few of the

shows that feature the use of math in solving crimes.

Pictured, right: Students in Lois Coles' math class use graphing calculators hooked into a navigation system to the teacher's laptop, which projects on a screen their progress on a math problem. In the background is eighth-grader Jarod LeFan.



So the neat tricks TV cops use, such as deblurring number plates and reconstructing accidents from skid marks, are not as far fetched as they seem. In the fight against crime for both TV cops and the real police force, the secret weapon is mathematics.

In this activity, you will be using math to solve a problem from the prime time **CBS** television show *Numb3rs.* 

### Algebra

Adapted from http://education.ti.com/go/NUMB3RS © 2006 Texas Instruments Incorporated

### NUMB3RS Activity: Meltdown

In "Harvest," Don and David discover a secret operating room in the basement of an old motel, which is being used to perform illegal kidney transplants. They find blood-soaked sheets and a pile of ice melting on a sheet of plastic in a corner. When Charlie sees the FBI's pictures, he notices that the size of the puddle formed by the melting ice depends on the time the picture was taken. He and Amita discuss how this information can be used to determine when the ice first started to melt. This will tell them when their suspects last used the operating room.

In this activity, we will assume that the ice is on a level surface, that it melts into a circular puddle of constant thickness, and that the room's temperature remains constant.

1. If the ice melts at a constant rate, what does that tell us about the rate at which the area of the puddle increases?

2. Use the formula A =  $\pi r^2$  to complete the following table for the area of a growing puddle. Leave your answers in terms of  $\pi$ .

Puddle Number	1	2	3	4
Radius	5 cm	10 cm	15 cm	20 cm
Area				

3. How does the area of the puddle increase when the radius increases from its original size by 5 cm, 10 cm, and 15 cm? Can you generalize the change in area for an increase of n cm?

### Algebra

Adapted from http://education.ti.com/go/NUMB3RS © 2006 Texas Instruments Incorporated

### NUMB3RS Activity: Meltdown

4. Algebraically, how much larger is  $(n + r)^2$  than  $r^2$ ? Compare this to your answers to #3.

Suppose Charlie has two pictures of the melting ice; the first one was taken at 8:45 A.M. and the second one was taken at 9:45 A.M. In the first picture, he determines the radius of the puddle to be 30 cm. In the second picture, it has grown to 32 cm.

5. What is the area in square centimeters  $(cm^2)$  that the puddle covered in each picture? What is the corresponding rate of increase in the area  $(cm^2/min)$ ? (Use 3.14 for  $\pi$ .)

6. When did the ice start to melt? (Hint: use the rate of increase in the area to find how long it took the puddle to grow to a radius of 30 cm.)

BENCHMARKS:	<b>MA.8.A.6.4</b> Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real world problems.
	<b>MA.8.G.5.1</b> Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)) and dimensions including temperature, area, volume, and derived units to solve problems.

### **CUT, FOLD, AND CONSTRUCT**

Adapted from M-DCPS's Geometry Measures Up Packet

Many students study surface area and wonder; *"When are we every going to need to know this?"* Yet, every year around the holiday winter season, millions of students eagerly await their wrapped presents under the tree never considering that their gift has a specific surface area. Without careful attention to the surface area, gift wrappers would waste a million miles of gift wrapping paper if they did not think carefully about the shape of each gift they wrapped.



Holiday gift giving began long before Christmas. The Romans would give gifts to one another on pagan festivals like Saturnalia, the winter solstice, and the Roman New Year. The tradition of gift giving became associated with Christmas because of the offerings of the Three Wise Men, though early on the Church discouraged the practice of gift giving because of its pagan associations. But by the Middle Ages the tradition had become so popular that it became a mainstay of the holiday season.

Early on gifts were wrapped in simple tissue paper or more sturdy brown paper. In the nineteenth century, gifts were sometimes presented in decorated cornucopias or paper baskets. Early gift wrappers had to be especially dexterous; scotch tape wasn't invented until 1930! And it wasn't until 1932 that the rolls of adhesive tape were sold in dispensers with cutter blades. Before then packages were tied up with string and sealing wax.

Innovations with gift wrap have continued. The 1980's introduced decorative plastic and paper gift bags, though these "new" bags weren't as new as some people thought. The Victorians had often given their gifts in decorated bags. The introduction of stick-on bows and cascade ribbons in the 80's and 90's further helped less than perfect gift wrappers. Nowadays one can wrap a gift without even using paper, by going on-line and sending an e-card wrapped in "virtual paper."

In this activity, you will be working with the surface areas of *Regular Polyhedrons*. You will cut out, fold, and construct the five *Regular Polyhedrons*.

### CUT, FOLD, AND CONSTRUCT

Adapted from M-DCPS's Geometry Measures Up Packet

#### ACTIVITY SHEET

A **Regular Polyhedron** is a solid, three-dimensional figure each face of which is a regular polygon with equal sides and equal angles. Every face has the same number of vertices, and the same number of faces meet at every vertex. An inscribed (inside) sphere touches the center of every face, and a circumscribed sphere (outside) touches every vertex.

There are five and only five of these figures, also called the Platonic Solids: the *tetrahedron, cube, octahedron, dodecahedron*, and *icosahedron*.

Before cutting out the figures, find the measurements of the sides and the heights of the triangles where appropriate. Write the measurements in the table below. If there are four sides that have the same dimensions, indicate the dimensions x 4.

Figure	Dimensions	Area of Base	Surface Area	Volume
CUBE				
ICOSAHEDRON				
OCTAHEDRON				
TETRAHEDRON				

Cut out each shape along the exterior sides. Decorate each shape. Fold along the interior segments. Assemble the solids by tucking in the tabs and gluing or taping. Display your solids by hanging them on a hanger or mounting them on a board (i.e., shoe box lid).



### **OCTAHEDRON**



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### **TETRAHEDRON**



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#### **ICOSAHEDRON**



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DODECAHEDRON



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#### Adapted from NCTM Journal 'mathematics teaching in the MIDDLE SCHOOL,' Oct. 2007

In mathematics and the arts, two quantities are in the *golden ratio* if the ratio between the sum of those quantities and the larger one is the same as the ratio between the larger one and the smaller. The *golden ratio* is approximately 1.6180339887.

At least since the Renaissance, many artists and architects have proportioned their works to approximate the *golden ratio*—especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the *golden ratio*—believing this proportion to be aesthetically pleasing. Mathematicians have studied the golden ratio because of its unique and interesting properties.

The **golden ratio** can be expressed as a mathematical constant, usually denoted by Greek letter  $\varphi$  (phi). The **golden ratio** is a special number that is derived mathematically from:

$$\frac{1+\sqrt{5}}{2}$$



The golden section is a line segment sectioned into two according to the golden ratio. The total length a+b is to the longer segment a as a is to the shorter segment b.

It is important not to confuse the **golden ratio** with the **Golden mean** (philosophy), the felicitous middle between two extremes, **Golden numbers**, an indicator of years in astronomy and calendar studies, or the **Golden Rule**.

In this activity, you will use proportional thinking to investigate the *golden* ration and see how approximations of the golden ratio are exhibited in parts of the human body.

Adapted from NCTM Journal 'mathematics teaching in the MIDDLE SCHOOL,' Oct. 2007

### **ACTIVITY SHEET**

### Investigating the Golden Ratio

Are we golden? Is the golden ratio somewhere in each of us?

Gather two to five friends and/or family members to help you fill out table 1.

#### Step 1:

Measure the height (B) and the navel height (N) of each person. Calculate the ratio B/N and record them in table 1.

#### Step 2:

Measure the length (F) of an index finger and the distance (K) from the fingertip to the big knuckle of each member of your group. . Calculate the ratio F/K and Record them in table 1.

#### Step 3:

Measure the length (L) of a leg and the distance (H) from the hip to the kneecap of each person. Calculate the ratio L/H and record them in table 1.

#### Step 4:

Measure the length (A) of an arm and the distance (E) from the fingertips to the elbow of each person. Calculate the ratio A/E and record them in table 1.

#### Step 5:

Measure the length (X) of a profile (the top of the head to the level of the bottom of the chin) and the length (Y) (the bottom of the chin) of each person. Calculate and record the ratio X/Y in the table 1.





Adapted from NCTM Journal 'mathematics teaching in the MIDDLE SCHOOL,' Oct. 2007

#### **ACTIVITY SHEET**

Example measurements:

	Express Each Ratio in Both its Fraction and Decimal Form					
Name	B/N	F/K	L/N	A/E	X/Y	
1. Paulette	$\frac{65}{39}$ or	$\frac{2.5}{1.5}$ or	$\frac{34}{17.3}$ or	$\frac{27}{10}$ or 2.7	$\frac{8.3}{4.5}$ or	
	1.666	1.66	1.97		1.84	
2. Kim	$\frac{72}{44}$ or	$\frac{2.8}{1.5}$ or	$\frac{36}{19}$ or 1.89	$\frac{27}{12}$ or 2.25	$\frac{8}{4}$ or 2.0	
	1.633	1.86				
3. Roy						
4. Ray						
5. Linda						

#### **USE TABLE 1 TO RECORD YOUR MEASUREMENTS**

### Table 1

	Express Each Ratio in Both its Fraction and Decimal Form					
Name	B/N	F/K	L/N	A/E	X/Y	
1.						
2.						
3.						
4.						
5.						

1. Which ratios in your table were close to the golden ratio and which were not?

2. Are you Golden? Explain why or why not.

Adapted from NCTM Journal 'mathematics teaching in the MIDDLE SCHOOL,' Oct. 2007

### **BENCHMARK:**

#### MA.8.A.6.1

Use exponents and scientific notation to write large and small numbers and vice versa and to solve problems.

### ANTI-DISCRIMINATION POLICY

### **Federal and State Laws**

The School Board of Miami-Dade County, Florida adheres to a policy of nondiscrimination in employment and educational programs/activities and strives affirmatively to provide equal opportunity for all as required by law:

Title VI of the Civil Rights Act of 1964 - prohibits discrimination on the basis of race, color, religion, or national origin.

**Title VII of the Civil Rights Act of 1964**, as amended - prohibits discrimination in employment on the basis of race, color, religion, gender, or national origin.

Title IX of the Educational Amendments of 1972 - prohibits discrimination on the basis of gender.

Age Discrimination in Employment Act of 1967 (ADEA), as amended - prohibits discrimination on the basis of age with respect to individuals who are at least 40.

**The Equal Pay Act of 1963**, as amended - prohibits gender discrimination in payment of wages to women and men performing substantially equal work in the same establishment.

Section 504 of the Rehabilitation Act of 1973 - prohibits discrimination against the disabled.

Americans with Disabilities Act of 1990 (ADA) - prohibits discrimination against individuals with disabilities in employment, public service, public accommodations and telecommunications.

**The Family and Medical Leave Act of 1993 (FMLA)** - requires covered employers to provide up to 12 weeks of unpaid, job-protected leave to "eligible" employees for certain family and medical reasons.

**The Pregnancy Discrimination Act of 1978** - prohibits discrimination in employment on the basis of pregnancy, childbirth, or related medical conditions.

**Florida Educational Equity Act (FEEA)** - prohibits discrimination on the basis of race, gender, national origin, marital status, or handicap against a student or employee.

Florida Civil Rights Act of 1992 - secures for all individuals within the state freedom from discrimination because of race, color, religion, sex, national origin, age, handicap, or marital status.

Veterans are provided re-employment rights in accordance with P.L. 93-508 (Federal Law) and Section 295.07 (Florida Statutes), which stipulates categorical preferences for employment.

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